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AN ANALYSIS OF NOSETIP BOUNDARY LAYER TRANSITION DATA

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AN ANALYSIS OF NOSETIP BOUNDARY LAYER TRANSITION DATA

by

M. L. FINSON

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SYMBOLS AND NOTATION

Primary Symbols

D	diameter of roughness element
k	roughness height (peak-to-valley)
ℓ	average spacing between roughness elements
M	Mach number
p	static pressure
R	radius
Re	Reynolds number
S	streamwise distance
T	temperature
u	velocity
x	streamwise distance
y	distance normal to wall
δ	boundary layer thickness
δ^*	boundary layer displacement thickness
θ	boundary layer momentum thickness
μ	absolute viscosity
ν	kinematic viscosity (μ/ρ)
ρ	density

Subscripts

∞	free-stream conditions
c	curvature
e	boundary-layer edge conditions
k	roughness height; conditions at $y = k$
m	geometrical mean of wall and edge conditions, e.g. $\nu_m = (\nu_e \nu_w)^{1/2}$

n	nose radius
o	stagnation conditions
t	conditions at transition location
w	wall conditions
θ	based on momentum thickness

I. INTRODUCTION

The prediction of boundary layer transition is an important consideration in the design of nosetips for advanced reentry vehicles. Turbulent heating is responsible for a major portion of the nosetip recession, and can also cause significant changes in nosetip shape that in turn degrade aerodynamic performance of the vehicle. Nosetip transition is a low altitude phenomenon, typically occurring below 25 km altitude. Due to the high Reynolds numbers, laminar stagnation point boundary layers are no more than a few mils in thickness. This dimension is not very large in comparison to the characteristic height (~ 1 mil) of the inherent surface roughness of candidate nosetip materials, and as a result roughness may be expected to play a dominant role in nosetip transition.

Several significant research programs have been carried out recently to identify the behavior of nosetip boundary layers under roughness-dominated conditions. The most extensive of these is probably the Passive Nosetip Technology (PANT) program performed by Aerotherm/Acurex Corp. under SAMSO support.[1] Extensive wind tunnel transition data were obtained on roughened calorimeter models for various values of roughness height, nose radius, total pressure, wall temperature, etc. The resulting transition locations have been correlated by Anderson[2] in terms of the Reynolds number based on momentum thickness at transition, the ratio of roughness height to momentum thickness, and the wall temperature ratio

$$Re_{e,\theta} = 215 \left(\frac{k}{\theta} \frac{T_e}{T_w}\right)^{-0.7} \qquad (1)$$

Other efforts under the PANT program have included the development of laminar and turbulent heat transfer correlations and the construction of computer codes for predicting nosetip recession and shape change.

Additional wind tunnel experiments are being conducted by Philco-Ford as part of the Advanced Penetration Problems (APP) program.[3] These tests will use roughened porous models, so that the combined effects of roughness and wall blowing can be simulated. Hot wire measurements of fluctuations within the boundary layer will also be obtained.

Equation 1 is not the only correlation that has been applied to nosetip transition. Another candidate is derived from the work of van Driest et al [4]

on the effect of an isolated row (at constant streamwise position) of spherical roughness elements on spheres and cones. They found that transition occurs at the location of the "trip spheres" if the "roughness Reynolds number" $Re_{k,k} = \rho_k u_k k/\mu_k$ exceeds a critical value. They also found that a correction factor based on k/R was required to reconcile the sphere results (R = sphere radius) with those for cones ($R = \infty$); this factor presumably represents the stabilizing influence of the centrifugal forces resulting from streamwise curvature. While this correlation was derived from experiments on isolated roughness, a very comparable expression has been suggested by van Driest,[5] Dirling,[6] and Swigart[7] for transition on nosetips with distributed roughness.

A quite different approach has been pursued by White.[8] A key parameter in his correlation is the "normalized vorticity number"

$$NVL = \left[\frac{\rho_o \mu_o}{\rho_\infty^2 u_\infty^2} \frac{du_e}{dx_o} \right]^{1/2} \qquad (2)$$

which is supposed to measure the effect of vorticity in the inviscid shock layer. White obtained a rather impressive correlation of the PANT data in the form

$$Re_{e,\theta} = \left[\left(\frac{T_e}{T_w} \frac{k}{\theta} \right)^{0.25} f\,(M_e) \right]^{g\,(NVL)} \qquad (3)$$

where $f\,(M_e)$ and $g\,(NVL)$ are empirically derived functions.

Finally, we might note several more fundamental theoretical approaches that have been developed to understand the nature of roughness-dominated nosetip transition. Wilcox[9] and the present author[10] have extended second-order turbulent closure models to transition, and Merkle et al.[11] have analyzed rough-wall transition in terms of linear stability theory. These models have shown a certain degree of agreement with the available transition data, and hopefully will be useful in elucidating the effect of various parameters.

This report describes the results of an attempt at performing a critical and independent evaluation of the existing ground test data on nosetip transition, and of the transition criteria used for nosetip design. The behavior of transition is examined as a function of parameters that are thought to be physically well-founded. An important aspect of this study

is an evaluation of the available data, and a determination of the manner in which the data base could be extended to provide a completely unambiguous definition of nosetip transition.

Our approach will be to list the various parameters that may reasonably be expected to influence nosetip transition, and then investigate the behavior of transition against these parameters. Ideally one should examine transition as a function of one parameter, with all other parameters fixed. However the data base may not be sufficiently extensive for this procedure to be followed with every potential parameter. It should be emphasized that this is intended to be a critical study of the available data. Although the author has participated in the development of fundamental models for the transition process, the present goal is not to validate such theories and we shall not rely upon them to any great extent. The theoretical background is, however, useful in identifying parameters, and it provides a means for estimating the effects of roughness on boundary layer properties which have not been considered in previous transition studies.

Certain qualifications should be recognized at the outset. This study is limited to nose region transition - transition that occurs before or slightly downstream of the sonic location; frustum transition is not considered. All of the relevant ground tests have surface roughness which is uniformly distributed and of the sand-grain or grit-blasted type. Wall blowing to simulate ablation has not been included in any ground tests to date. Furthermore, most of the data have been obtained on models with hemispherical noses; although some tests have involved nose shapes that could result from laminar ablation, none have been performed on the highly complicated "gouged" surfaces that result from transitional or turbulent heating. Also, all the ground tests are at free-stream Mach numbers of six or less, for which real gas effects are small.

As a matter of common sense, it should be recognized that the "ideal" or "true" correlation is not necessarily the one with the least data scatter. The parameters involved in a transition correlation must truly control transition, and one should not artifically adjust the parameters to reduce the apparent data scatter (e.g. by taking a fractional power). The true data scatter can be determined only by considering trajectory characteristics, as will be described below.

A final point to be born in mind is that any simple correlation in terms of the local conditions at transition may not be completely adequate. The transition process is quite complicated and generally involves a finite distance over which disturbances are amplified. If this amplification distance is appreciable, a "perfect" correlation based on local conditions should not be expected, but consideration of non-location conditions would probably over-complicate an already difficult problem.

II. TRANSITION PARAMETERS

Let us now examine the various parameters that could control nosetip transition. The basic principle of dimensional analysis states that the non-dimensionalized dependent variable (transition) should be a function of the several independent variables, each of which is also non-dimensionalized. One could attempt a formal dimensional analysis of transition in terms of such basic flight parameters as free-stream properties, etc., but we know that boundary layer edge properties will be the relevant quantities and we may as well proceed with that in mind.

Table I is a reasonably inclusive list of the parameters that might affect nosetip transition. Anderson and Bartlett have presented a similar list.[12] The table indicates physical effects and the corresponding non-dimensional parameters. In some cases there is a choice of several dimensionless quantities to measure a single effect. For example, the dependent variable - the occurrence of transition - may be defined by distance to transition (S_t) or by the boundary layer thickness at the transition location (θ_t or δ_t^*), and may be non-dimensionalized by the nose radius (R_n) or as a Reynolds number. Some thought will be required to choose the most appropriate of such possibilities.

Several of the parameters listed in Table I may be expected to be less important than others, and shall not be considered in the data analyses to follow. One such quantity is the freestream disturbance level. Atmospheric fluctuation velocities are low and, as already mentioned, it is expected that surface roughness is the dominant source of fluctuations in reentry situations.* To be sure, freestream disturbances are always present in wind tunnels and may affect the wind tunnel data obtained at relatively small roughness. In such cases the issue will be to suggest ways in which uncontaminated data may be obtained in other types of experiments.

The unit Reynolds number (Re/ft) is another parameter that will be disregarded. We cannot accept this quantity because it is fundamentally dimensional. A wide variety of wind tunnel transition data shows a clear dependence on

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*Furthermore, in those flight cases where atmospheric fluctuating velocities are not completely negligible, the associated length scales would be so large that ambient fluctuations would be ineffective for triggering transition.

TABLE I

Potential Transition Parameters

	Effect	Parameter
Dependent:	Transition	S_t/R_n or $Re_{e,S}$ or $Re_{e,\theta}$
Independent:	Roughness	k/θ or $Re_{e,k}$ or $Re_{k,k}$
	Wall Cooling	T_w/T_e
	Mach Number	M_e
	Ablation	$\rho_w v_w/\rho_e u_e$
	Pressure Gradient	$\frac{\theta^2}{\nu_e} \frac{du_e}{dx}$
	Longitudinal Curvature	θ/R_c
	Freestream Disturbances	u_e'/u_e
	Vorticity Interaction	$NVL = \left[\rho_o \mu_o \frac{du_e}{dx}\right]^{1/2} \Big/ \rho_\infty u_\infty$
	Streamline Divergence	$\frac{\theta}{R} \frac{dR}{dx}$
	Unit Reynolds Number	No proper quantity

unit Reynolds number, but this must occur because some other parameter such as freestream disturbance level depends on unit Reynolds number or on a Reynolds number based on tunnel size. Indeed, Pate and Schueler have demonstrated a strong connection between boundary layer transition and the aerodynamic noise radiated from turbulent boundary layers on the wind tunnel walls.[13]

It is well-known that compressibility has an important effect on transition. However, nosetip transition generally occurs within the subsonic region; for the PANT tests typical values of the edge Mach number at the transition location are $M_e = 0.3 - 0.5$, and the highest values are $M_e \sim 0.7$. At these low values it seems doubtful that compressibility effects should be significant. As a result, the edge Mach number M_e will not be retained as a fundamental parameter. It should, however, be recognized that other parameters, such as the temperature ratio T_w/T_e, are coupled to the Mach number.

We are not aware of any definitive studies that would indicate how streamline divergence might affect transition. As the transition location approaches the stagnation point, one might imagine that the behavior of transition could be altered by the diverging nature of the flow there. But here again we have an effect that should be small; for the transition locations observed in the wind tunnel tests, the boundary layer is already quite thin compared to the distance from the axis ($\theta/R \leq 2 \times 10^{-3}$).

Finally there is the effect of shock layer vorticity, as measured by the normalized vorticity number defined in Eq. 2. For a spherical nose the modified Newtonian expression for the pressure distribution may be used to show that

$$NVL^2 = \frac{\rho_o \mu_o}{\rho_\infty^2 u_\infty^2}\left(\frac{du_e}{dx}\right)_o = \sqrt{\frac{\rho_o}{\rho_\infty}}\,\frac{\mu_o}{\mu_\infty}\,\sqrt{2\frac{p_o}{\rho_\infty u_\infty^2}}\,Re_{\infty,R_n}^{-1} \sim Re_{\infty,R_n}^{-1} \quad (4)$$

Thus, the normalized vorticity number is closely related to the Reynolds number based on freestream properties and nose size. Our laminar boundary layer calculations indicate that the shock layer vorticity effect should be quite small for cases of interest. Due to the high Reynolds numbers involved, the boundary layers are thin and there should be very little swallowing of vorticity or entropy.* Nevertheless, White obtained a

* Entropy swallowing would be more important for nonspherical nose shapes such as a conical nose with a small spherical region.

very impressive correlation of the form indicated by Eq. (3). One disturbing aspect of this correlation is that it involves the edge Mach number, which we have already argued to be unimportant in the subsonic region. Even more disturbing is the fact that the NVL correlation probably does not contain significant information on transition, but rather represents an approximate correlation for any laminar boundary layer on a spherical nose (before or after transition). Baker has reached a similar conclusion.[14] Anderson and Bartlett have shown that a laminar boundary layer solution can fall within 5% of the NVL correlation curve nearly all the way around the nose, even for a non-transitional case.[12]

In light of these arguments, the parameters from Table I that remain under consideration are the dependent variable (transition) and surface roughness, wall cooling, ablation, pressure gradient, and longitudinal curvature as independent variables. Several dimensionless numbers are possible for some of these effects, as indicated in the table, because more than one length scale is available. As a measure of transition, at least four length scales come to mind: the distance from the stagnation point to the transition location S_T, and three potential measures of the boundary layer thickness at transition - the boundary layer thickness δ, the displacement of thickness δ^*, and the momentum thickness θ. The latter two are defined in the usual manner

$$\delta^* = \int_0^\infty \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy, \quad \theta = \int_0^\infty \frac{\rho u}{\rho_e u_e} \left(1 - \frac{u}{u_e}\right) dy \tag{5}$$

Of course for given nose shape, wall temperature, etc., these four length scales are not independent of each other. Nevertheless, it is appropriate to seek the length that is most relevant to the transition process. If transition is even approximately a local process, the flow distance S will not be particularly relevant. The boundary layer thickness δ cannot be considered seriously, since it is arbitrarily defined from some point in the tail of the velocity profile (e.g. where $u/u_e = 0.99$ or 0.995) and has little physical significance. And, the displacement thickness is not particularly appropriate because it is sensitive to the wall temperature ratio (even negative for very cold walls). Hence, δ^* can be a measure of the wall temperature as much as a measure of the "width" of the boundary layer. None of these objections apply to the momentum thickness θ and we conclude, as have most other investigators, that θ is the most relevant measure of the boundary layer thickness. The corresponding dimensionless quantity should be a Reynolds number Re_θ. The alternate choice θ/R makes little sense, since it is doubtful that the nose radius should be very relevant - particularly if one

considers nonspherical nose shapes such as flat or conical noses. It remains to be determined whether the fluid properties ρ, u, and μ should be evaluated at the boundary layer edge or at some point within the boundary layer in computing Re_θ.

Some discussion is also in order regarding the optimal measure of the surface roughness. First of all, it is assumed that the roughness can be characterized by a single height k. Surface roughness generally involves a distribution of roughness element heights, shapes, and spacings. No single parameter can suffice for all types of surfaces, particularly if the ablated surfaces of composite materials are considered. However, the wind tunnel tests performed to date involve a narrow range of roughness character - particles of a single size brazed onto a smooth surface, or grit - blasted surfaces. Either method yields a surface that is probably similar to sand-grain roughness or to the roughness inherent to a material such as graphite. The experimental data are presented against the "peak-to-valley" height. This definition, while not terribly precise, is probably adequate for the present purposes and will be used henceforth. The theory described in Ref. 10 indicates that the bulk of the fluctuations created by a roughness element originate in the vicinity of the peak, so that the peak-to-valley height should be more appropriate than the average, rms, or equivalent sand-grain heights.

As discussed in the Introduction, prior correlations of nosetip transition data have utilized either the "roughness Reynolds number" $Re_{k,k}$ or the ratio of roughness height to momentum thickness k/θ.* The relative merits of these two parameters have been the subject of some debate. To a large extent the choice between $Re_{k,k}$ and k/θ is rather a moot point, because the two are related. A correlation in terms of one can be transformed into a correlation involving the other, although $Re_{e,\theta}$ and T_w/T_e are also involved in such a transformation. If k/θ is greater than about five, conditions at $y = k$ will be close to the edge conditions, and

$$Re_{k,k} \cong Re_{e,\theta} \frac{k}{\theta} \tag{6}$$

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* Flat plate data are generally presented in terms of $Re_{e,k}$. This parameter is reasonable for flat plates, since roughness elements must protrude beyond the boundary layer edge sufficiently near the leading edge. However, for a nose region the stagnation point boundary layer has a finite thickness, and the elements need not protrude to the edge, making $Re_{e,k}$ less relevant.

Conversely, if k/θ is small ($k/\theta < 2 - 3$), the tops of the elements will be in the linear portion of the velocity profile near the wall so that

$$Re_{k,k} \sim \frac{\nu_e}{\nu_k} \, Re_{e,\theta} \left(\frac{k}{\theta} \right)^2 \tag{7}$$

Here the viscosity ratio will be a function of T_w/T_e and also k/θ. Thus in general

$$Re_{k,k} = Re_{e,\theta} \frac{k}{\theta} \, f\,(k/\theta,\ T_w/T_e) \tag{8}$$

On a physical basis, Anderson[2] has argued that the roughness parameter should provide a measure of the strength of the disturbances introduced by the roughness. The fluctuation energy introduced by an element should be proportional to the mean flow conditions at the element, $\rho_k u_k^2/\rho_e u_e^2$ which is indeed a function of k/θ and T_w/T_e, the parameters used in that study.* However this argument is not unique. It presumes that the quantity of interest is the relative fluctuation energy produced by the elements. What may be more appropriate is the rate at which energy is produced by elements compared to the rate at which it is dissipated by viscous effects or diffuses to the surface. Indeed, in calculations based on the model of Ref. 10, it was typically found that the source and sink terms are nearly in balance; the fluctuations grow or decay according to whether the source terms are slightly greater than or slightly less than the sum of the dissipative and diffusive terms. If the fluctuations produced behind an element are proportional to the local mean velocity ($u'^2 \sim u^2$), if the width of the element is proportional to the element height so that the mass flux per unit surface area is $\rho_k u_k k$, and if the element spacing is proportional to k so that there are k^{-2} elements per unit surface area, then the rate of production of fluctuations energy per unit volume is

$$\sim u_k^2 \; (\rho_k u_k k) \; k^{-2} = \rho_k u_k^3/k \tag{9}$$

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* One might have to modify this argument to account for the fact that there could be a minimum Reynolds number below which no disturbances are introduced, by analogy with the cut-off Reynolds number below which von Karman vortice are not shed behind a cylinder in a uniform stream. If so, $Re_{k,k}$ would enter. However little is known about such a minimum Reynolds number for the production of fluctuations behind elements of a distributed surface roughness field.

And, if we further assume that the length scale of the fluctuations is proportional to the element height, the dissipation per unit volume is

$$\sim \mu\, u_k^2/k^2 \tag{10}$$

Dividing the expression in Eq. (9) by that in Eq. (10) yields the roughness Reynolds number $Re_{k,k}$ as the ratio of the rate at which energy is produced to that at which it is dissipated. Equation (10) would also hold for the rate of viscous diffusion to the (bottom of the) wall, so that $Re_{k,k}$ also describes the ratio of production to diffusion.

The reader will probably recognize that these theoretical arguments are inconclusive regarding whether k/θ or $Re_{k,k}$ is the more fundamental parameter because there are several possible ways to normalize the rate at which fluctuations are introduced by roughness elements. Our experience with the theory of Ref. 10 leads us to prefer $Re_{k,k}$ but even that conclusion is not unambiguous. In any case, k/θ and $Re_{k,k}$ are interrelated and one can always transform from one to the other. Perhaps a more relevant argument in favor of using $Re_{k,k}$ in a transition correlation is that the results are more sensitive to $Re_{k,k}$. As will be shown later, the transformation from a plot of Re_θ vs $Re_{k,k}$ to a plot of Re_θ vs k/θ involves taking approximately the 3/8 power of both the ordinate and abscissa. While the data scatter will be greater when plotted against $Re_{k,k}$, this method of plotting will exhibit more sensitivity to other effects such as wall temperature ratios and longitudinal curvature. It is primarily for this reason that we shall proceed in terms of $Re_{k,k}$.

The wall temperature is known to have an important effect on transition. The general result from wind tunnel experiments on flat plates or cones and from linear stability calculations is that reducing the wall temperature stabilizes the boundary layer (see, for example, Ref. 15). However there have been numerous observations of the opposite trend or "transition reversal" (decrease in transition Reynolds number with decrease in wall temperature) under various conditions. In the range $0.5 \leq T_w/T_e \leq 1.0$, Dunlap and Kuethe[16] observed very little effect of wall temperature on transition location on polished hemispheres. With single-element roughnesses on cone, though, van Driest and Boisin[17] observed a transition reversal. The PANT calorimeter tests confirm that nosetip transition falls in the regime of transition reversal. As time progressed during each test run, the wall temperature increased from 300°K to 500°K, typically, while the freestream and edge conditions remained essentially constant. With increasing wall temperature, the transition location was observed to recede from the stagnation point, implying increasing values of $Re_{e,\theta}$ with

increasing T_w/T_e. This behavior undoubtedly results from the variation of fluid properties (density and viscosity) with temperature across the boundary layer. In the classical "nonreversal" regime (increasing transition Reynolds number with decreasing wall temperature), the mean velocity profile is altered by the fluid property variation in such a manner as to alter the linear stability characteristics. However, no such explanation has been accepted for the reversal regime, and it is safe to say that the mechanism by which the wall temperature affects transition is not well understood. In any case, the ratio of wall temperature to edge temperature should provide an adequate measure of the temperature effect.* As we shall see below, the use of fluid properties in the "middle" of the boundary layer rather than at the outer edge describes the dependence of nosetip transition on wall temperature in a simple and adequate manner.

Finally, there are the effects of pressure gradient and streamwise curvature. For any convex nose shape the pressure gradient is favorable, which tends to stabilize the boundary layer. The centrifugal forces due to surface curvature are also stabilizing for a convex shape. As indicated in Table I, a quantity similar to the Polhausen parameter $(\delta^2/\nu)\, du_e/dx$ may be used as a nondimensional measure of the pressure gradient. Classical linear stability calculations by Schlichting and Ulrich (see Ref. 18) for low speed flow indicate the pressure gradient effect to be important if $\theta^2\,(du_e/dx)/\nu_e$ is greater than about 0.02. For cases of interest this parameter may be as large as 0.07. Centrifugal effects have also been studied in connection with the flow between concentric cylinders. Here, stability calculations[18] indicate an important effect in low speed flow if $\theta/R \geqslant 0.003$, a value which occurs in nosetip boundary layers at Reynolds numbers of interest.

It is rather difficult to distinguish between the effects of pressure gradient and curvature in analyzing nosetip transition data. With the Newtonian approximation, the pressure gradient is proportional to the surface curvature. Each parameter is proportional to θ/R_c, where R_c is the local radius of curvature in the streamwise direction (equal to the nose radius for a hemispherical nose). To be sure, the pressure gradient parameter contains an additional factor of Re_θ, but Re_θ does not vary by an extremely large factor in the transition data. For simplicity, we shall measure the combined effects of curvature and pressure gradient by θ/R_c. We expect that the transition Reynolds number Re_θ may tend to increase for larger values of θ/R_c. This is somewhat of a departure from previous studies, which have used k/R_c as a measure of the importance of curvature and have

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* The total temperature or recovery temperature might be more appropriate than the edge temperature, but the difference is small for the low edge Mach numbers of interest here.

modified $Re_{k,k}$ rather than Re_θ. The present approach is based on the expectation that curvature and pressure gradients should stabilize the boundary layer - that is, raise the value of Re_θ required for transition - rather than reduce the effective roughness height.

III. ANALYSIS OF WIND TUNNEL NOSETIP TRANSITION DATA

In light of the discussion of the previous section, we expect that nosetip transition may be expressed in the following functional form:

$$Re_\theta = f\,(Re_{k,k},\ T_w/T_e,\ \theta/R_c) \tag{11}$$

It should be remembered that the wall temperature ratio is expected to enter through the transport properties ρ and μ, and in Eq. (11) we have not yet specified where these properties are to be evaluated in computing Re_θ.

A laminar boundary layer solution is required to determine Re_θ, $Re_{k,k}$, and θ/R_c in Eq. (11). Previous investigators have employed smooth wall boundary layer computations, but estimates for the effect of roughness on boundary layer development will be incorporated here. The model upon which these estimates are based is described in Ref. 10. Basically, it is presumed that the flow around individual roughness elements is attached and parallel to the mean surface. Drag on the elements tends to decelerate the mean velocity and is described by a distributed sink term for $y \le k$ that is added to the momentum equation:

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\ \mu \frac{\partial u}{\partial y} - \frac{1}{2}\rho u^2 C_D D/\ell^2 \tag{12}$$

Here D (y) is the element width at height y ($y \le k$) and ℓ is the interelement spacing (assumed uniform). For the calculations to be presented here, the elements were assumed to be hemispheres to obtain D(y), C_D was 0.6, and ℓ was 0.2 k. This representation was found to yield a realistic description of transitional and fully turbulent rough-wall boundary layers,[10] but there are hardly any measurements to confirm its accuracy for laminar flows. The consequence of this model is that boundary layer thicknesses are increased for very rough walls, $k/\theta \gtrsim 4$. Figure 1 shows solutions as a function of distance around the nose for a typical case, from which it may be seen that the rough-wall solution differs negligibly from the smooth-wall solution for $k/\theta \le 2$, while there is an appreciable effect for large roughness. This effect is important for the wind tunnel tests with largest roughness.

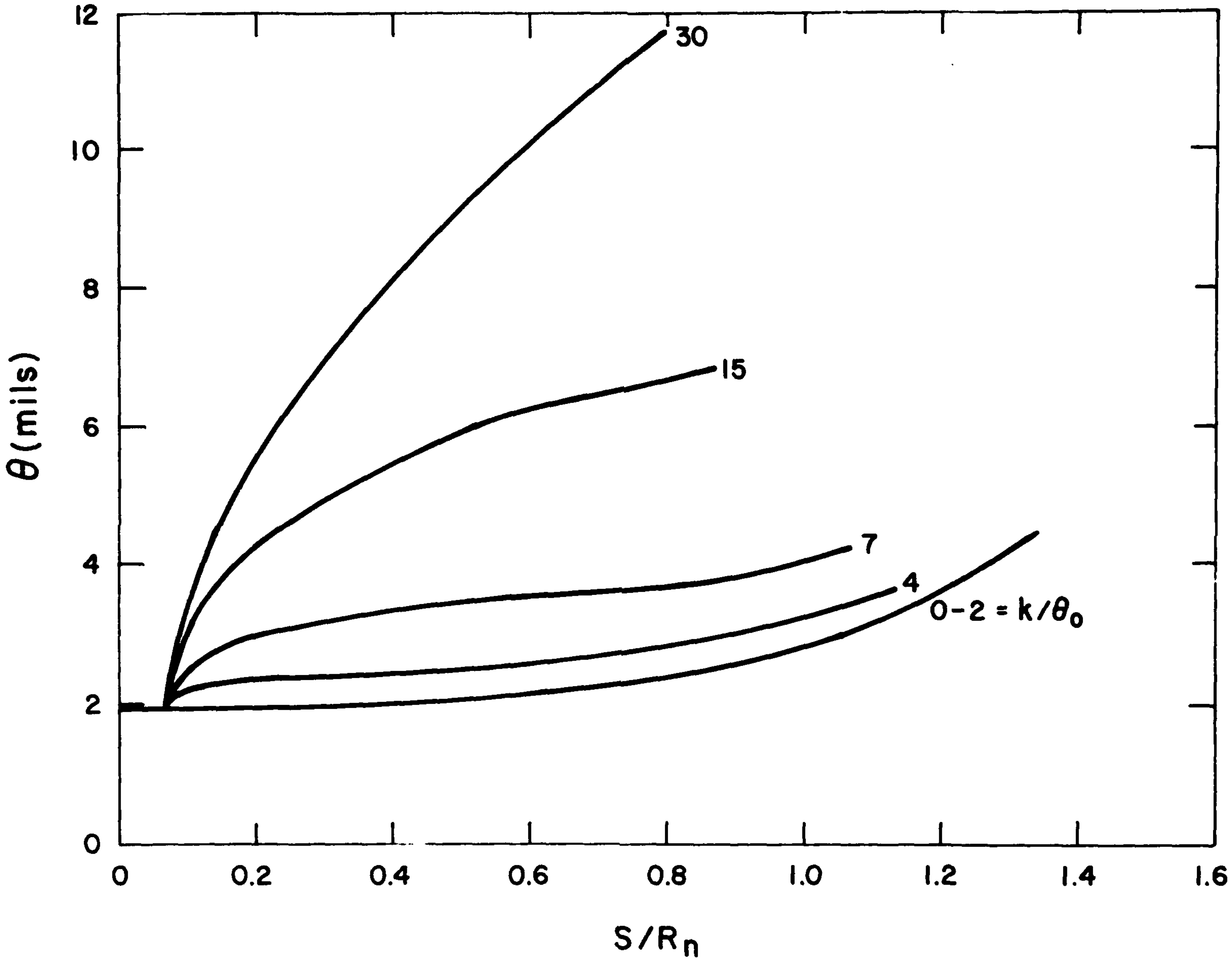

Fig. 1 Effect of Roughness on Computed Laminar Boundary Layer Thickness.

One of the first things that we investigated was the influence of wall temperature. In Fig. 2 we plot the momentum thickness Reynolds number at transition based on edge conditions vs. roughness Reynolds number for the PANT data. We separated the data according to ranges of wall temperature. Figure 2 clearly shows the trend of increasing $Re_{e,\theta}$ with increasing wall temperature. As already mentioned, this effect may be expected to result from the temperature dependence of density and viscosity. Hence, we next considered the use of conditions (ρ, μ) at some point within the boundary layer in computing Re_θ at transition. If one uses the wall values for ρ and μ (but continues to use u_e), it is found that the trend shown in Fig. 2 is reversed - that is, the wall temperature effect is over-corrected. The fact of the matter is that the kinematic viscosity is very sensitive to temperature ($\nu \sim T^{1.7}$) and varies substantially across a boundary layer. This is illustrated in Fig. 3, where we plot ν / ν_e across the boundary layer for $T_w/T_e = 0.44$. Note that ν varies by a factor of four across the boundary layer for this temperature ratio. We would claim, rather intuitively, that the most relevant value for the viscosity (that which really controls the boundary layer development) would be a value that is appropriate to the "middle" of the boundary layer. The edge value, ν_e, applies only in the outermost part of the boundary layer, and the wall value is appropriate only to a narrow region very near the wall. As a measure of the viscosity in the middle of the boundary layer, we suggest the geometrical mean of the wall and edge values*

$$\nu_m = \sqrt{\nu_w \nu_e} \tag{13}$$

In Fig. 4 we show the PANT transition data replotted in terms of

$$Re_{m,\theta} = u_e \theta / \nu_m \tag{14}$$

The data are identified by three different symbols according to low, intermediate, and high values of the wall temperature ratio. Except perhaps

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*One could also consider the viscosity at the height $y = \theta$ or $y = \delta^*$, or at the height where $u/u_e = 0.5$. However, the geometrical mean defined by Eq. 13 gives comparable results and is far more convenient to apply to the analysis of transition data. To be logically self-consistent, one might also consider use of the velocity at the same location in computing Re_θ, but such a velocity would essentially be a constant fraction of u_e and hence would not materially alter the nature of any correlation of data.

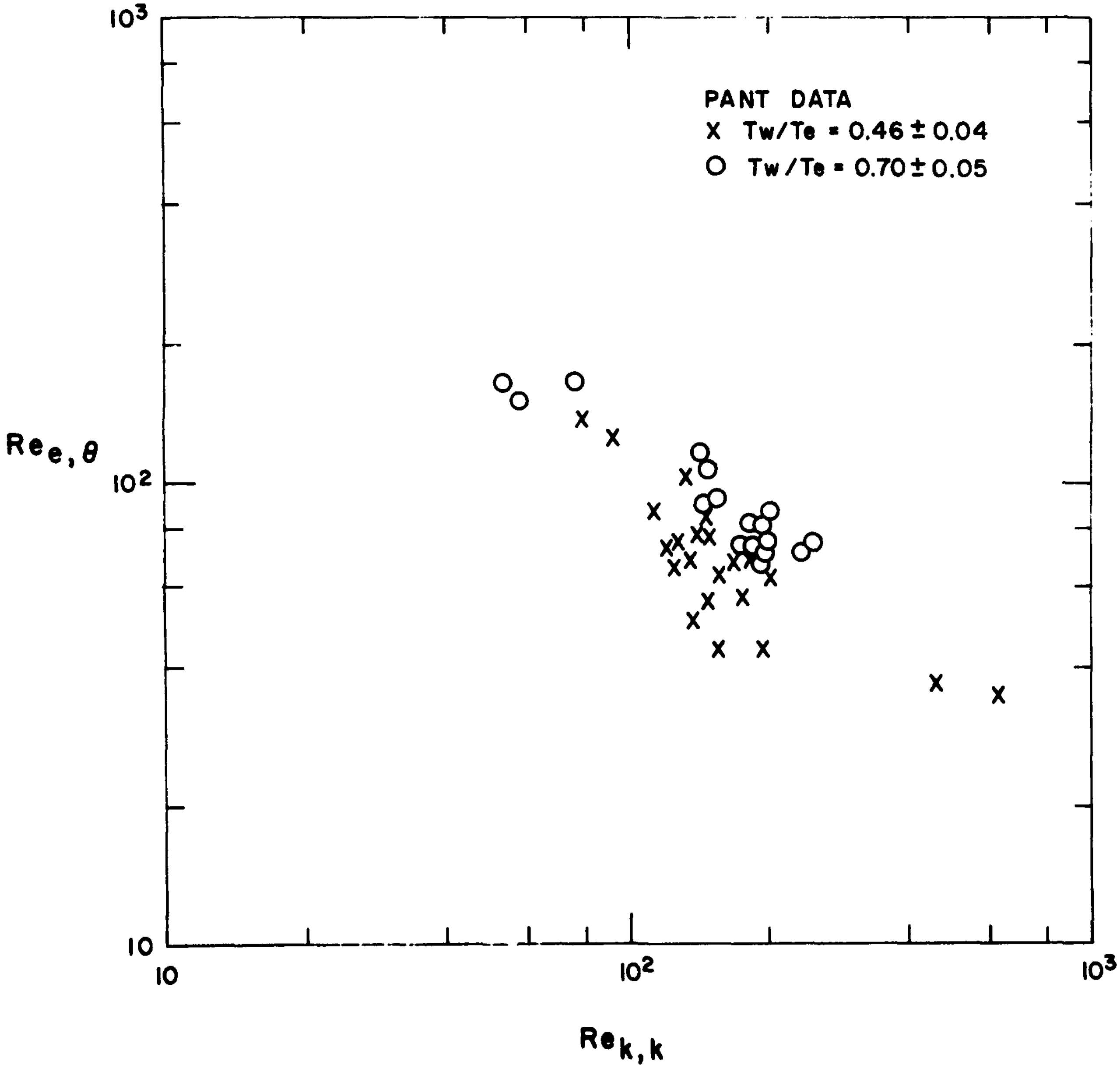

Fig. 2 Effect of Wall Temperature on Transition Reynolds Number (data from Ref. 2).

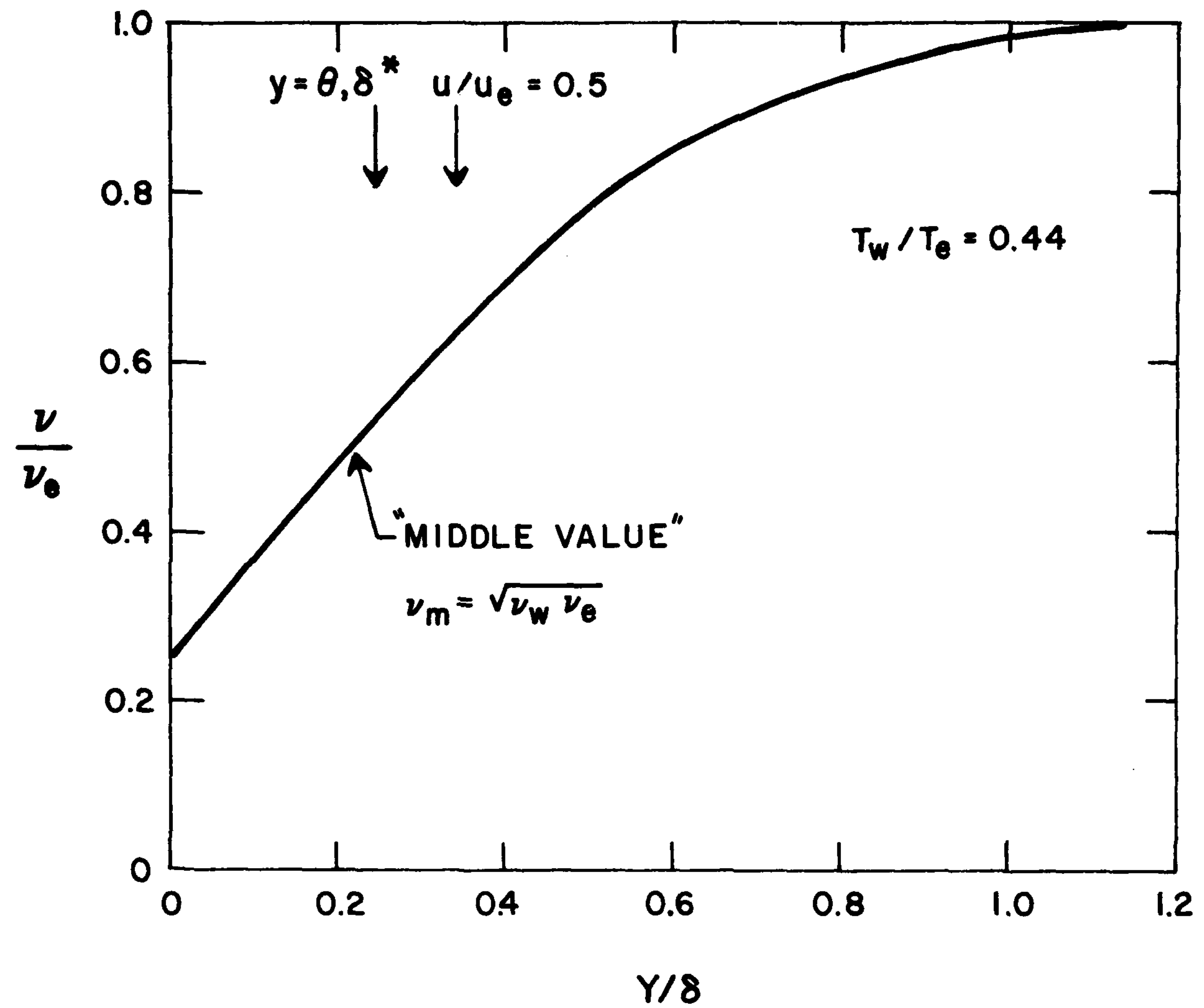

Fig. 3 Variation of Kinematic Viscosity Across a Laminar Boundary Layer with $T_w/T_e = 0.44$.

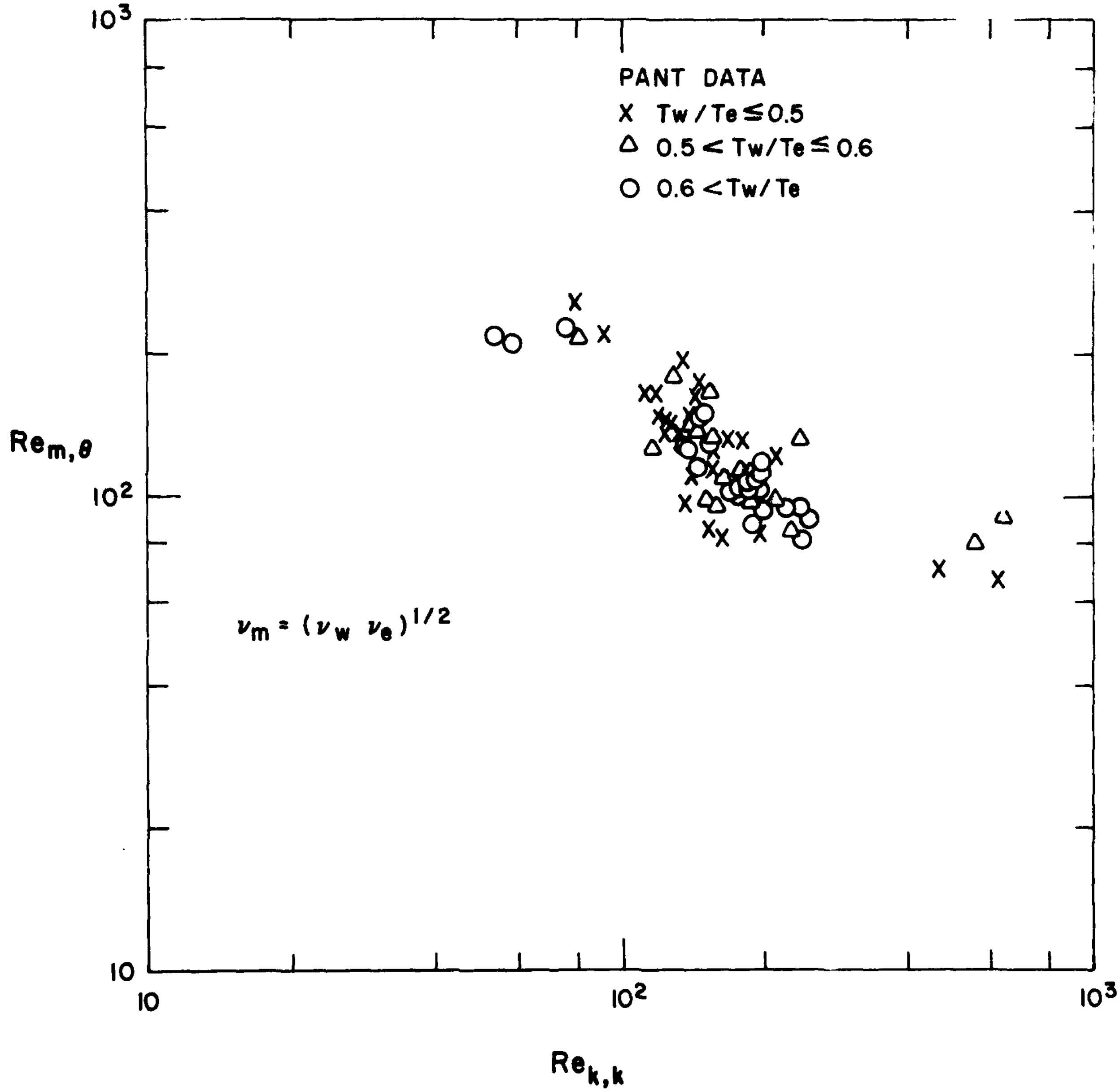

Fig. 4 Transition Reynolds Number Based on Viscosity in the "Middle" of the Boundary Layer vs. Roughness Reynolds Number (data from Ref. 2).

for the few data points at very large roughness, no significant dependence on wall temperature is discernible. Thus, the use of the viscosity within the boundary layer provides a straightforward manner for explaining the wall temperature effect. As already mentioned, the dependence of transition upon wall temperature under conditions of "transition reversal" is not well understood. Whether this simple concept applies to more general situations remains to be demonstrated.

It is not possible to define the effect of curvature/pressure gradient in such a satisfactory manner. Ideally, one should study the variation of transition Reynolds number with θ/R_c, with all other parameters (namely, $Re_{k,k}$) held constant. This procedure, however, cannot be followed to any extent with the available data. The difficulty is that there tends to be a positive correlation between the values of θ/R_c and $Re_{k,k}$ in the cases tested in the PANT program. As a result it is difficult to separate the effects of roughness and curvature/pressure gradient in the data. The cases run at small roughness heights are generally run at high freestream Reynolds number (otherwise there would be no transition), in which case θ/R_c will be small. The reverse holds for large roughness. To some extent this trend can be modified by changes in nose size or shape, but sufficient variations have not yet been investigated. However, observations of transition "onset" may provide additional clues regarding the role of θ/R_c.

In Fig. 5 we illustrate the cross-correlation between θ/R_c and $Re_{k,k}$ in the PANT data. The several points plotted at $\theta/R_c = 10^{-4}$ correspond to conical noses (60^o half-angle), for which $R_c = \infty$ and which should really be plotted at $\theta/R_c = 0$. Note that there is hardly any value of $Re_{k,k}$ at which there were tests conducted over a wide range of θ/R_c. The only potentially interesting value of $Re_{k,k}$ would be about 150. But, as will become clear shortly, the curvature effect is appreciably only for $\theta/R_c \geqslant 10^{-3}$. There are only two cases with $\theta/R_c \geqslant 10^{-3}$ at $Re_{k,k} \cong 150$, and that is not much with which to work. Figure 6 shows the PANT transition data, with special attention called to the conical cases. These cases involve no curvature and tend to fall below the others, but only if the plot is viewed with sufficient imagination.

We considered a curvature correction to the transition Reynolds number $Re_{m,\theta}$ of the form $(1 + C\,\theta/R_c)^{-1}$. If the constant C is much less than 10^3, this correction term has little effect. On the other hand, if C is 2000 or greater, the correction term causes the spherical nose data to fall below the data for the conical nose. The resulting plot of the PANT data with C = 1000 is shown in Fig. 7. The conical nose

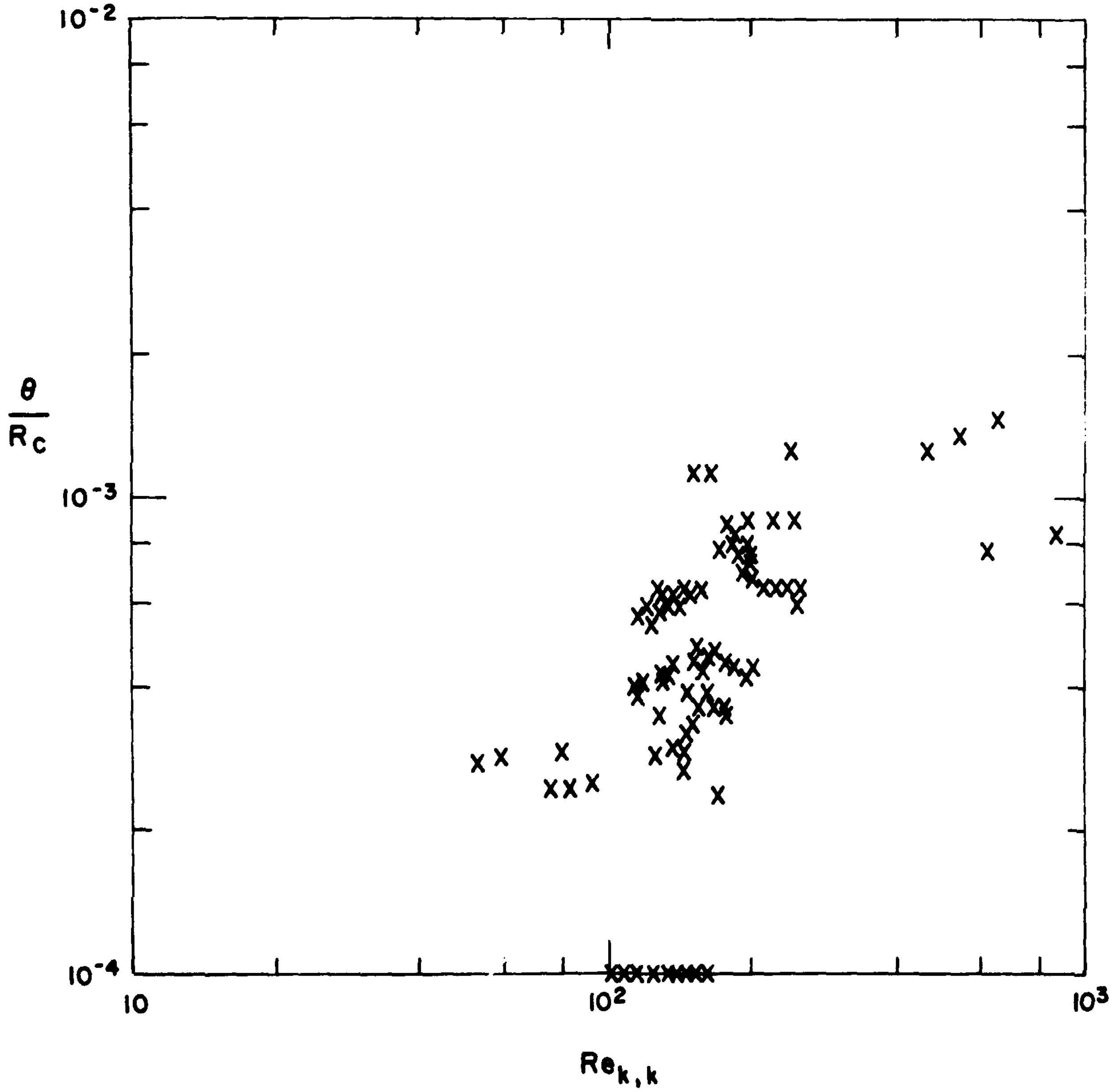

Fig. 5 Range of Conditions Covered by PANT Tests.[2]

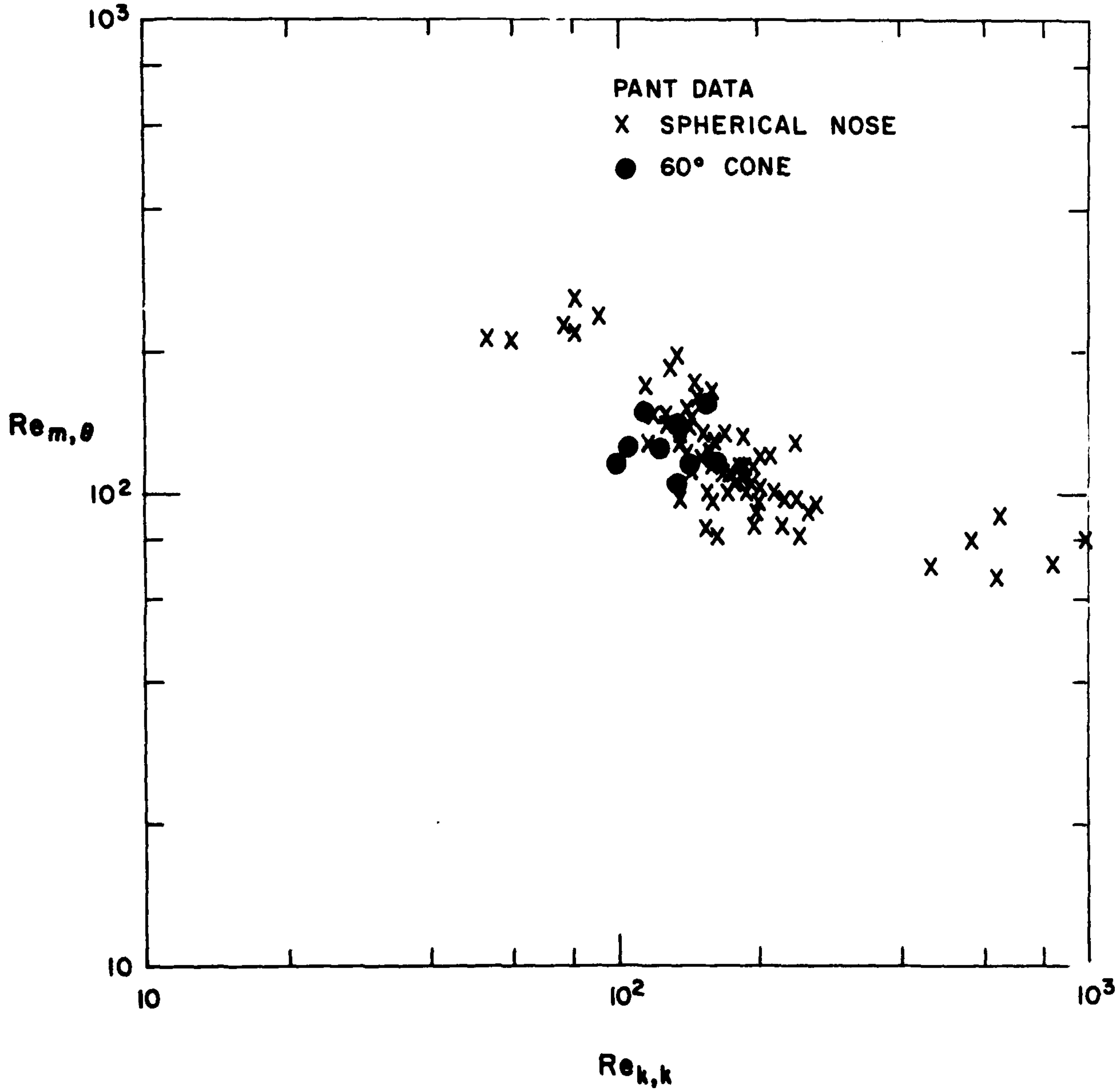

Fig. 6 Comparison of Transition Reynolds Numbers for Hemispherical and Conical Noses (data from Ref. 2).

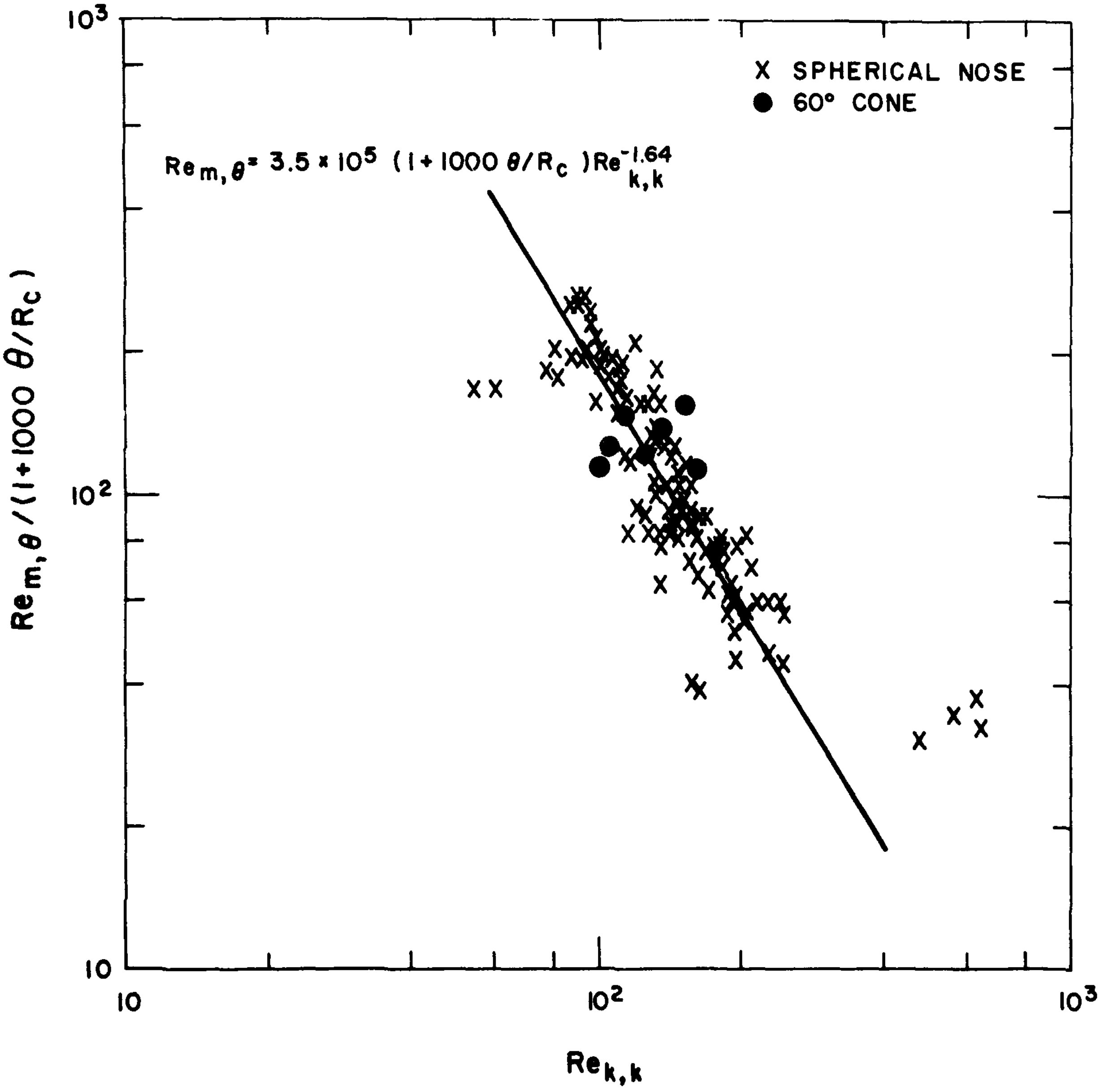

Fig. 7 PANT Transition Data[2] Corrected for the Effect of Curvature.

results are now rather well aligned with those for spherical noses. A straight line has been drawn through the bulk of the data:

$$Re_{m,\theta} = 3.5 \times 10^5 (1 + 1000\ \theta/R_c) Re_{k,k}^{-1.64} \tag{15}$$

It should be emphasized that this equation represents only a tentative correlation of the available data. The extent of the curvature effect is not well defined, and this uncertainty translates into some uncertainty in the exponent of $Re_{k,k}$. Also it should not be expected that this straight line could be extended to indefinitely large or small values of $Re_{k,k}$. Freestream noise will tend to dominate the effect of roughness in wind tunnel tests at small roughness heights, and one might even admit the possibility of a different behavior in flight as $Re_{k,k}$ becomes small. The few tests with very large roughness suggest that there may be a minimum Re_θ below which transition will not occur, at least if one accounts for the effect of roughness on the laminar boundary layer, as has been done here.

There are various other observations of nosetip transition, which we have not discussed thus far. Of these, none spans the range of conditions tested in the PANT series. And, with the exception of the recent Advanced Penetration Problems tests conducted by Philco-Ford at AEDC,[3] the quality of the transition data is much less due to poorer resolution of the transition location and less careful characterization of the surface roughness. Figure 8 compares other sources of nosetip transition data with the PANT data. Bearing in mind the qualifications just mentioned, there is reasonable agreement between the different sources. It should be noted that the APP data extend to lower values of the roughness Reynolds number than do the PANT data, but the APP data for $Re_{k,k} \lesssim 80$ are possibly influenced by tunnel noise. The sounding rocket data are potentially interesting because they involve actual flight results, but must be qualified by extremely approximate characterization of the roughness. Also, they may be influenced by body vibrations during the powered portions of the flights.

Finally, let us discuss the issue of transition "onset". The original PANT correlation of Eq. (1) was a necessary but not sufficient condition for transition. In that study it was found that if the maximum value of $Re_{e,\theta} \left(\frac{k}{\theta}\ \frac{T_e}{T_w}\right)^{0.7}$ only slightly exceeded the value of 215 (or was less than 255, to be precise), transition would not occur. This maximum value occurs in the vicinity of the sonic location. In Fig. 9 we investigate the need for a separate onset criterion with the present method of plotting the data. The coordinates are the same as in the previous two figures. The curves

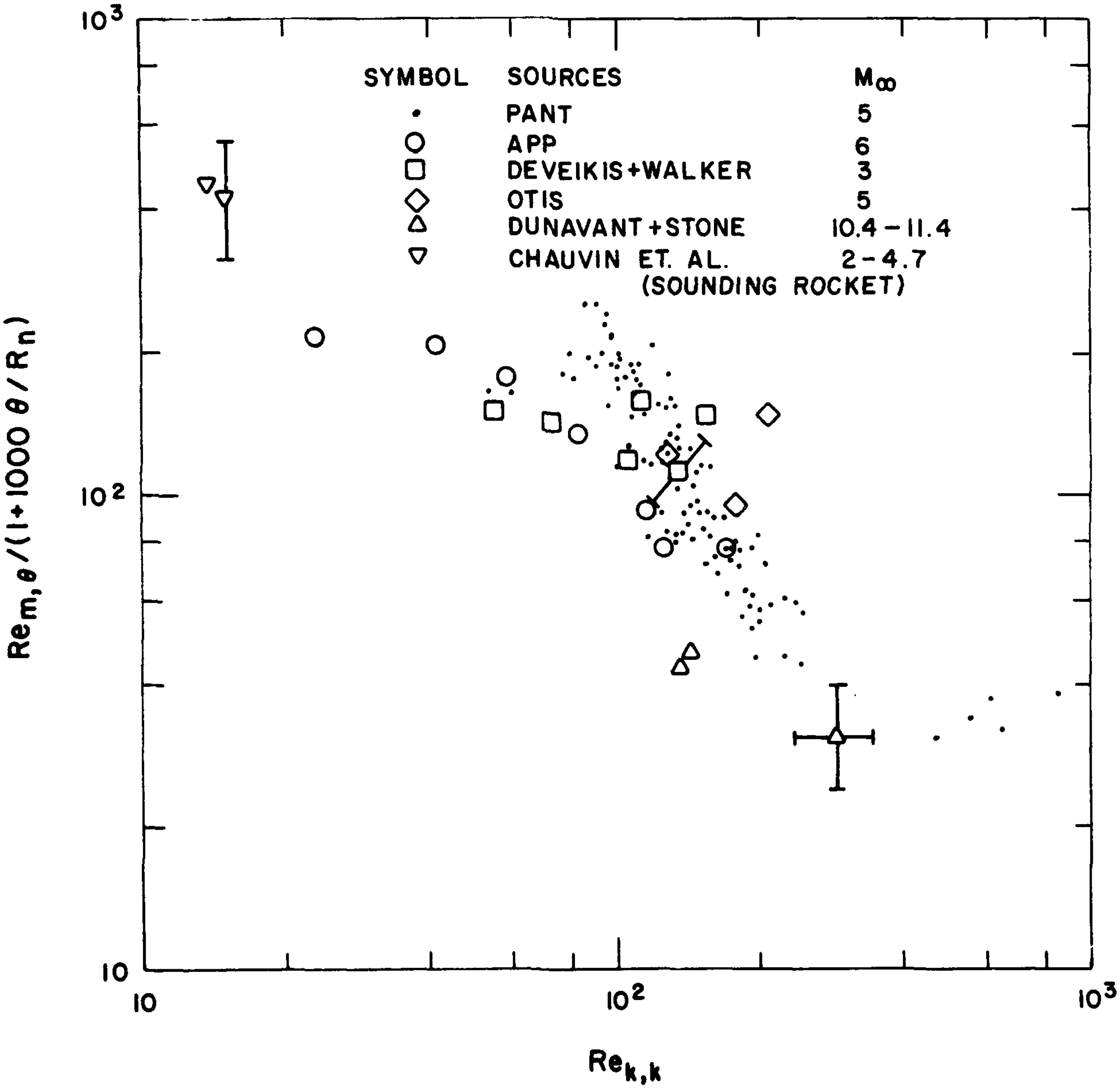

Fig. 8 Comparison of PANT Transition Data[2] with those from other Studies.[3,20-26]

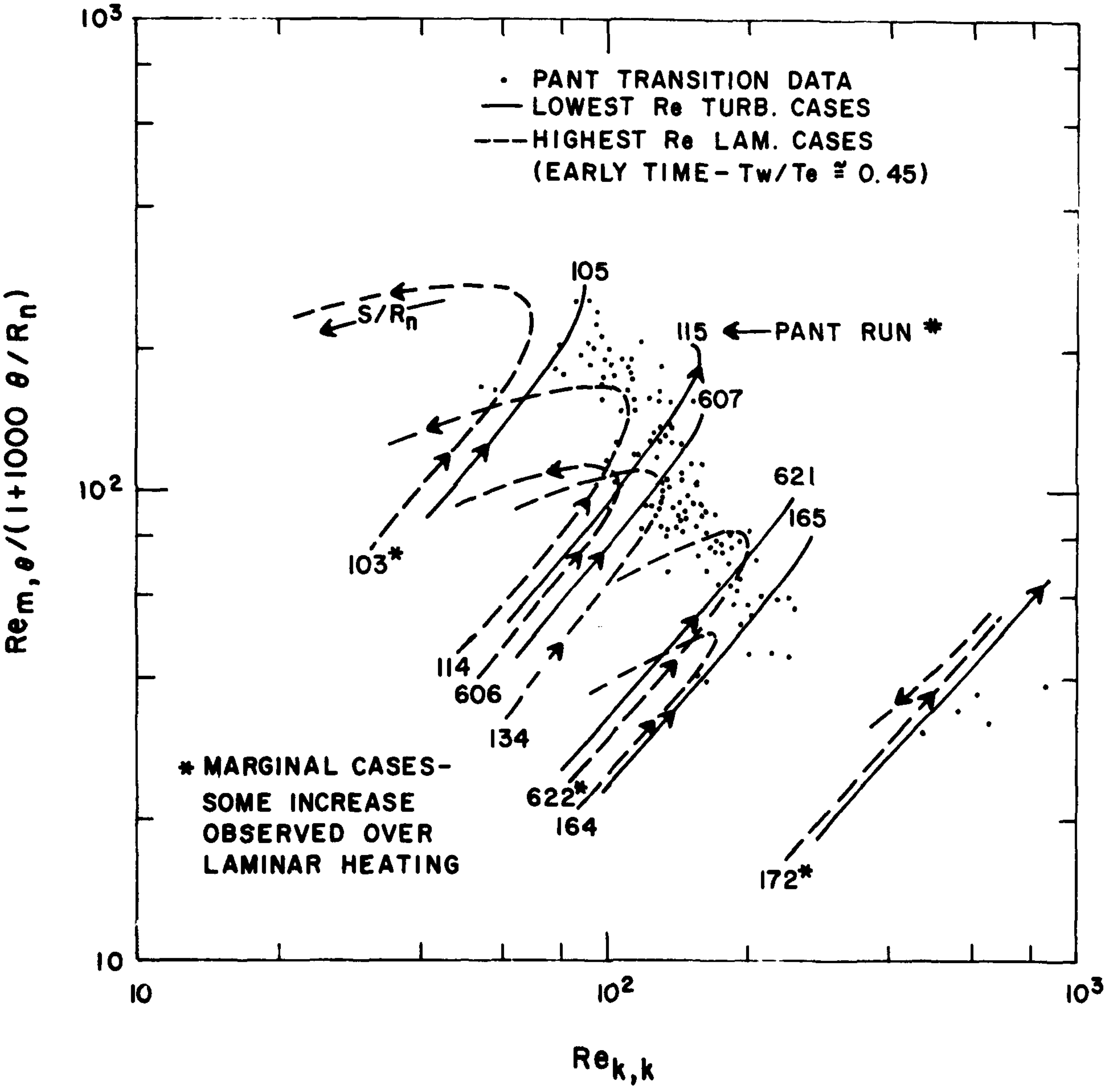

Fig. 9 Trajectories of Various PANT Tests on Transition Map. Each Curve is the Locus of Points for Increasing Values of S/R_n.

are "trajectories" for individual PANT runs, with distance around the nose varying along each curve. The trajectories are drawn for early times in the runs ($t = 1$ sec) when the wall temperatures are low; the PANT run numbers are indicated for each curve. The dashed curves correspond to cases which remained laminar, and they are the laminar cases that have the highest Reynolds numbers and are closest to transition. The solid lines indicate cases that become turbulent, and these cases are generally those with the lowest Reynolds numbers that still experienced transition. For sake of clarity, we did not extend the trajectories of the turbulent cases very far around the body; they would have shapes similar to those of the laminar cases, only displaced upwards and to the right. Of the laminar cases, the very rough case, #172, appears to extend above the transition data; one other, #622, extends to the top of the transition zone. None of the others reaches the middle of transition data zone. It is further noted that three cases (103, 622, 172) exhibited "transitional" heating distributions - heat transfer rates between laminar and fully turbulent values. With the exception of the case at very large roughness, where there is very little data, there appears to be no fundamental problem with "onset" in the present approach. The curvature effect is important here, since it has a greater effect at lower Reynolds numbers. With the curvature correction used in this study, the highest Reynolds number laminar cases generally remain below the mean of the transition data.* It is somewhat reassuring that this is so, since Anderson has speculated that the curvature effect was responsible for the need for a separate onset criterion in addition to the PANT transition criterion.[2]

The present manner of plotting the available nosetip transition data differs in several regards from those of previous studies. Most notably, the PANT correlation is based on k/θ rather than $Re_{k,k}$, and the present treatment of the curvature effect is unique. To a large extent the current data are not adequate to differentiate between the various approaches in a statistically significant manner. This is due to the difficulty in separating the effects of roughness and curvature discussed already. In the final section some tests are suggested that should resolve this issue. With regard to the difference between k/θ and $Re_{k,k}$ as fundamental independent parameters, there is of course a transformation between the two, as was discussed in Section II. Equations (6) and (7) describe this transformation, depending on whether k/θ is large or small. In either case we may write

$$Re_{k,k} = Re_{m,\theta}\, f(k/\theta, T_w/T_e) \tag{16}$$

* This conclusion should be verified for the data at later times, which have not been published in detail.

Inserting this into the result derived here, Eq. (15), we obtain

$$Re_{m,\theta}^{2.64} = f(k/\theta,\ T_w/T_e, \theta/R_c) \tag{17}$$

This illustrates the extent to which the coordinates used in this study are expanded in comparison to the PANT coordinates: by the power of 2.64!

IV. DISCUSSION AND RECOMMENDATIONS

As indicated at the outset, the goal of this effort is to specify what is known as well as what cannot yet be determined from the data currently available. The roughness Reynolds number $Re_{k,k}$ has been used here to correlate the data. This parameter may be somewhat more relevant than the non-dimensional roughness height k/θ, and transition is more sensitive to $Re_{k,k}$. To describe the effect of wall temperature ratio (T_w/T_e) we followed the intuitive concept that transition should be governed by fluid properties somewhere in the middle of the boundary layer and introduced $\nu_m = (\nu_w \nu_e)^{1/2}$, the geometrical mean of the wall and edge values of the viscosity. Finally, the value of Re_θ required for transition was modified by a factor involving θ/R_c to account for the effect of streamwise curvature and/or pressure gradient. This factor offers an explanation for the transition onset phenomenon.

However, the unresolved issues are probably more noteworthy than the ones resolved. First of all, as discussed in Sec. III, the curvature effect is poorly defined by the existing data. It is difficult to separate the effects of curvature and roughness. This difficulty explains the various exponents of the roughness height k and various treatments of the curvature terms that have appeared in the literature. Additional data are required to settle this matter. Interesting cases would be a conical nose at relatively large roughness or hemispheres of smaller R_n than those tested to date. Specific conditions to supplement the PANT tests would include a 60° fore-cone at $k \geq 10$ mil and a hemisphere with $R_n \lesssim 0.75$ in, $k \cong 3$ mil.*

An equally serious deficiency exists for small values of the roughness height, say $k/\theta \leqslant 1$ or $Re_{k,k} \leqslant 100$. Typical graphite nosetip materials have $k \leqslant 0.5$ mil and fall into this regime in reentry. Tunnel noise may influence the corresponding calorimeter transition data. Since freestream noise is difficult to eliminate in any practical wind tunnel, it would seem imperative that tests in other types of facility, such as a ballistic range, be carried out to confirm the behavior of transition at relatively small roughness. There are further advantages of ballistic range experiments, as will be discussed below.

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* Tests at the latter conditions were attempted during PANT Series A but no useful transition data were obtained, apparently due to the effects of axial heat conduction in the calorimeter models.

The behavior at very large roughness is also not well-defined, although of less interest for current design considerations. There were only two PANT tests with $k/\theta > 10$ (each yielded a data point at three different values of the wall temperature). The relative uncertainty in transition location is quite large; for these cases, the transition distances are comparable to the thermocouple spacing (it would be a good idea if error bars were identified in the future). The straight line drawn through the bulk of the data in Fig. 7 cannot be expected to be reliably extrapolated to very large roughness values - there is nothing sacred about a power-law behavior with most physical processes. Indeed, there is no reason to expect transition to occur at $Re_\theta \to 0$ as $k \to \infty$, at least if one accounts for the effect of roughness on the laminar boundary layer growth as has been done here. One might even anticipate the limiting behavior to approach a constant value of Re_θ or $Re_\theta / (1 + 1000\ \theta/R_c)$ as $k \to \infty$. Additional calorimeter tests with better streamwise resolution would clarify the nature of transition at large roughness.

A whole host of additional questions arise when one considers predicting flight transition with a correlation based on wind tunnel data. Ablation occurs on actual nosetips in reentry, but there is no mass addition in the calorimeter tests studied above. It is commonly observed that blowing destabilizes the boundary layer on a smooth wall, but one could speculate that a moderate amount of blowing might tend to "lubricate" a rough wall. Basic experiments, for example wind tunnel tests on porous, roughened calorimeter models, would clarify this issue.

A very practical question in relating calorimeter data to flight situations regards the proper manner of defining the roughness height k for actual nosetip materials. The "peak-to-valley" height used to characterize the calorimeter roughness is a sensible but intuitive quantity. On the other hand, detailed statistical measurements are often made on materials such as graphite (such measurements are best obtained on surfaces that have been previously exposed to simulated laminar heating). No such measurements were performed on the surfaces of the PANT models. Transition is sufficiently sensitive to roughness height that it is important to determine precisely how the "peak-to-valley" height is related to statistical properties of the roughness. This issue becomes even more critical if one considers composite weave materials, such as carbon-carbon, in which case it may be necessary to introduce additional parameters measuring the element spacing and shape to properly characterize the surface.

Finally, ballistic range experiments with potential nosetip materials would be invaluable in bridging the gap between wind tunnel tests and actual reentry conditions. Except for heating history, the ballistic range offers an almost perfect simulation of reentry. Potential nosetip materials can be tested, in the presence of laminar ablation (subject, of course, to limitations in heating history), at reentry Mach numbers, and in the probable absence of facility-related effects such as tunnel noise. Pyrometric techniques have been developed to detect surface temperature profiles, from which transition location can be inferred.[19]

Table II summarizes the various experiments that would resolve the issues outlined here and lead to a reliable capability for predicting nosetip boundary layer transition in flight situations.

TABLE II

Potential Nosetip Transition Experiments

Issue	Suggested Experiments
Small Roughness	Ballistic Range
Large Roughness	Wind Tunnel Calorimeter Tests with Better Spatial Resolution
Blowing	Wind Tunnel Calorimeter Tests with Porous, Roughened Models
Curvature/Pressure Gradients	Calorimeters - 60° Cones with $k = 10$ mil; Hemispheres with $R_C \leqslant 0.75''$, $k = 3$ mil
Composite Materials	Ballistic Range
Simulation of Flight	Ballistic Range

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